Time : 5 Hour

STD 11 Maths

kd 90+ ch-2 relation and functions

Total Marks : 220

* **Choose the right answer from the given options. [1 Marks Each]** **[40]**

1. $f(x)=\sqrt{9-x^2}$. Find the range of the function:

(A) R (B) R+ (C) [-3, 3] (D) [0, 3]

Ans. :

4. [0, 3]

Solution:

We know, square root is always non-negative.$\sqrt{9-x^2}>0$.

So, the range of the function is set of positive real numbers from 0 to 3.

2. The domain of the function $f(x)=\sqrt{2-2x-x^2}$ is:

(A) $\left[-\sqrt{3},\sqrt{3}\right]$ (B) $\left[-1,-\sqrt{3},-1+\sqrt{3}\right]$

(C) $\left[-2,2\right]$ (D) $\left[-2-\sqrt{3},-2+\sqrt{3}\right]$

Ans. :

b. $\left[-1,-\sqrt{3},-1+\sqrt{3}\right]$

Solution:

$f(x)=\sqrt{2-2x-x^2}$

Since, $2-2x-x^2\geq 0$

$x^2+2x-2\leq 0$

$\Rightarrow x^2-2x-2+1-1\leq 0$

$\Rightarrow (x-1)^2-\left(\sqrt{3}\right)^2\leq 0$

$\Rightarrow \left[x-\left(1-\sqrt{3}\right)\right]\left[x-\left(1+\sqrt{3}\right)\right]\leq 0$

$\Rightarrow \left(-1-\sqrt{3}\right)\leq x\leq(-1+\sqrt{3})$

Thus, domain $(f)=\left[-1-\sqrt{3},-1+\sqrt{3}\right]$

3. Let n(A) = m and n(B) = n, Then, the total number of non-empty relations that can be defined from A to B is:

(A) mn (B) -1mn (C) 2mn -1 (D) 2mn -1

Ans. :

d. 2^{mn} -1

4. The range of $f(x)=\frac{1}{1-2\cos x}$ is:

(A) $\left[\frac{1}{3},1\right]$ (B) $\left[-1,\frac{1}{3}\right]$

(C) $\left(-\infty,-1\right)\cup\left[\frac{1}{3},\infty\right)$ (D) $\left[-\frac{1}{3},1\right]$

Ans. :

b. $\left[-1, \frac{1}{3}\right]$

Solution:

We know that $-1 \leq \cos x \leq 1$ for all $x \in R$

Now,

$-1 \leq \cos x \leq 1$

$\Rightarrow -1 \leq \cos x \leq 1$

$\Rightarrow -2 \leq -2\cos x \leq 2$

$\Rightarrow -1 \leq 1 - 2\cos x \leq 3$ (Adding 1 ro each term)

But,

$\cos x \neq \frac{1}{2}$

$\Rightarrow 1 - 2\cos x \in \left[-1, 3\right] - \{0\}$

$\Rightarrow \frac{1}{1-2\cos x} \in \left(-\infty, -1\right] \cap \left[\frac{1}{3}, \infty\right)$

$\therefore$ Range of $f(x) = \left(-\infty, -1\right] \cap \left[\frac{1}{3}, \infty\right)$

Disclaimer: The range of the function does not matches with either of the given options. The range matches with option (c) if it is given as $\left(-\infty, -1\right] \cap \left[\frac{1}{3}, \infty\right)$

5. If $f(x) = \frac{x - 1^3}{x^3}$ then $f(x) + f\left(\frac{1}{x}\right)$ is equal to:

(A) $2x^3$

(B) $\frac{1}{x^3}$

(C) 0

(D) 1

Ans. :

c. 0

6. If $f(x) = \frac{\sin^4 x + \cos^2 x}{\sin^2 x + \cos^4 x}$ for $x \in R$, then f(2002) =

(A) 1

(B) 2

(C) 3

(D) 4

Ans. :

a. 1

Solution:

Given,

$f(x) = \frac{\sin^4 x + \cos^2 x}{\sin^2 x + \cos^4 x}$

On dividing the numerator and denominator by $\cos^4 x$, we get

$f(x) = \frac{\tan^4 x + \sec^2 x}{1 + \tan^2 x \sec^2 x}$

$= \frac{1 + \tan^4 x + \tan^2 x}{1 + \tan^2 x(1 + \tan^2 x)}$

$= \frac{1 + \tan^4 x + \tan^2 x}{1 + \tan^4 x + \tan^2 x} = 1$ (For every $x \in R$)

For $x = 2002$,

We have,

$f(2002) = 1$

7. $f(x) = \sqrt{9 - x^2}$. Find the domain of the function:

(A) (0, 3) (B) (0, 3) (C) (-3, 3) (D) (-3, 3)

Ans. :

c. (-3, 3)

Solutuion:

We know radical cannot be negative.

So, $9 - x^2, \geq 0$

$(3 - x)(3 + x) \geq$

$\Rightarrow (x - 3)(x + 3) \leq 0$

$\Rightarrow x \in [-3, 3]$.

8. If set A has 2 elements and set B has 3 elements then how many subsets does A × B have?

(A) 6 (B) 8 (C) 32 (D) 64

Ans. :

d. 64

Solution:

If set A has m elements and set B has n elements then A × B has m × n elements.

We know, a set has 2^r subsets if it has r number of elements.

Here, A × B has 2 × 3 = 6 elements. So, number of subsets of A × B will be 2^6 i.e. 64.

9. If $3f(x) + 5f\left(\frac{1}{x}\right) = \frac{1}{x} - 3$ for all non-zero x, then f(x) =

(A) $\frac{1}{14}\left(\frac{3}{x} + 5x - 6\right)$

(B) $\frac{1}{14}\left(-\frac{3}{x} + 5x - 6\right)$

(C) $\frac{1}{14}\left(-\frac{3}{x} + 5x + 6\right)$

(D) None os these.

Ans. :

d. None os these.

Solution:

$3f(x) + 5f\left(\frac{1}{x}\right) = \frac{1}{x} - 3 \ldots (i)$

Multiplying (1) by 3,

$15f\left(\frac{1}{x}\right) + 9f(x) = \frac{3}{x} - 9 \ldots (ii)$

Replacing x by $\frac{1}{x}$ in (i)

$3f\left(\frac{1}{x}\right) + 5f(x) = x - 3$

Multiplying by 5

$15f\left(\frac{1}{x}\right)+25f(x)=5x-15\ldots(iii)$

Solving (ii) and (iii),

$-16f(x)=\frac{3}{x}-5x+6$

$\Rightarrow f(x)=\frac{1}{16}\left(-\frac{3}{x}+5x-6\right)$

Disclaimer: The question in the book has some error, so, none of the options are matching with the solution. The solution is created according to the question given in the book.

10. If $2f(x)-3f\left(\frac{1}{x}\right)=x^2(x\neq 0)$, then f(2) is equal to:

(A) $-\frac{7}{4}$ (B) $\frac{5}{2}$ (C) -1 (D) None of these.

Ans. :

a. $-\frac{7}{4}$

Solution:

$2f(x)-3f\left(\frac{1}{x}\right)=x^2\ldots(i)\ \ (x\neq 0)$

Replacing x by $\frac{1}{x}$

$2f\left(\frac{1}{x}\right)-3f(x)=\frac{1}{x^2}\ldots(ii)$

Solving equations (i) & (ii)

$-5f(x)=\frac{3}{x^2}+2x^2$

$\Rightarrow f(x)=\frac{-1}{5}\left(\frac{3}{x^2}+2x^2\right)$

Thus, $f(2)=\frac{-1}{5}\left(\frac{3}{4}+2\times 4\right)$

$=\frac{-1}{5}\left(\frac{3+32}{4}\right)$

$=-\frac{7}{4}$

11. Choose the correct answers:

The domain of the function f given by $f(x)=\frac{x^2+2x+1}{x^2-x-6}$.

(A) R – {3, –2} (B) R – {–3, 2} (C) R – [3, –2] (D) R – (3, –2)

Ans. :

a. R – {3, –2}

Solution:

Given that: $f(x)=\frac{x^2+2x+1}{x^2-x-6}$

f(x) is defined if $x^2-x-6\neq 0$

$\Rightarrow x^2-3x+2x-6\neq 0$

$\Rightarrow (x-3)(x+2)\neq 0$

$\Rightarrow x\neq -2, x\neq 3$

So, the domain of f(x) = R - {-2, 3}

12. If $\mathrm{f(x)} = \log\left(\frac{1+x}{1-x}\right)$ and $\mathrm{g(x)} = \frac{3x+x^3}{1+3x^2}$, then f(g(x)) is equal to:

(A) f(3x) (B) {f(x)}3 (C) 3f(x) (D) -f(x)

Ans. :

c. 3f(x)

Solution:

$\mathrm{f(x)} = \log\left(\frac{1+x}{1-x}\right)$ and $\mathrm{g(x)} = \frac{3x+x^3}{1+3x^2}$

Now,

$\frac{1+g(x)}{1-g(x)} = \frac{1+\frac{3x+x^3}{1+3x^2}}{1-\frac{3x+x^3}{1+3x^2}}$

$= \frac{1+3x^2+3x+x^3}{1+3x^2-3x-x^3}$

$= \frac{(1+x)^3}{(1-x)^3}$

Then, $\mathrm{f(g(x))} = \log = \log\left(\frac{1+g(x)}{1-g(x)}\right)$

$= \log\left(\frac{1+x}{1-x}\right)^3$

$= 3\mathrm{f(x)}$

13. The range of the function: $\mathrm{f(x)} = \sqrt{(x-1)(3-x)}$:

(A) (-1, 1) (B) (-1, 1) (C) (-3, 3) (D) (-3, 1)

Ans. :

a. (-1, 1)

14. The domain of definition of $\mathrm{f(x)} = \sqrt{\frac{x+3}{(2-x)(x-5)}}$ is:

(A) $(-\infty, -3] \cup (2, 5)$ (B) $(-\infty, -3] \cup (2, 5)$

(C) $(-\infty, -3] \cup [2, 5]$ (D) None of these.

Ans. :

a. $(-\infty, -3] \cup (2, 5)$

Solution:

$\mathrm{f(x)} = \sqrt{\frac{x+3}{(2-x)(x-5)}}$

For f(x) to be defined,

$(2-x)(x-5) \neq 0$

$\Rightarrow x \neq 2, 5 \ldots (i)$

Also, $\frac{(x+3)}{(2-x)(x-5)} \geq 0$

$\Rightarrow \frac{(x+3)(2-x)(x-5)}{(2-x)^2(x-5)^2} \geq 0$

$\Rightarrow (x+3)(x-2)(x-5) \leq 0$

$\Rightarrow x \in \left(-\infty, -3\right] \cap (2, 5) \ldots (ii)$

From (i) and (ii)

$x \in \left(-\infty, -3\right] \cup (2, 5)$

15. If A = {1, 2, 3}, B = {1, 4, 6, 9} and R is a relation from A to B defined by 'x' is greater than y. The range of R is

(A) {1, 4, 6, 9} (B) {4, 6, 9} (C) {1} (D) none of these.

Ans. :

c. {1}

Solution:

A = {1, 2, 3} and B = {1, 4, 6, 9}

R is a relation from A to B defined by: x is greater than y.

Then R = {(2, 1), (3, 1)}

∴ Range (R) = {1}

16. If A = {1, 2, 4}, B = {2, 4, 5}, C = {2, 5}, then (A - B) × (B - C) is:

(A) {(1, 2), (1, 5), (2, 5)} (B) {(1, 4)}

(C) (1, 4) (D) none of these.

Ans. :

b. {(1, 4)}

Solution:

A = {1, 2, 4}, B = {2, 4, 5} and C = {2, 5}

(A - B) = {1}

(B - C) = {4}

So, (A - B) × (B - C) = {(1, 4)}

17. Let R be a relation on N defined by x + 2y = 8. The domain of R is:

a. {2, 4, 8}
b. {2, 4, 6, 8}
c. {2, 4, 6}
d. {1, 2, 3, 4}

Ans. :

c. {2, 4, 6}

Solution:

x + 2y = 8

⇒ x = 8 - 2y

For y = 1, x = 6

y = 2, x = 4

y = 3, x = 2

Then R = {(2, 3), (4, 2), (6, 1)}

∴ Domain of R = {2, 4, 6}

18. If the set A has p elements, B has q elements, then the number of elements in A × B is:

a. p + q

b. p + q + 1

c. pq

d. p^2

Ans. :

c. pq

Solution:

n(A × B) = n(A) × n(B)

n(A × B) = p × q = pq

19. If R is a relation from a finite set A having m elements of a finite set B having n elements, then the number of relations from A to B is:

a. 2^{mn}

b. $2^{mn} - 1$

c. 2mn

d. m^n

Ans. :

a. 2^{mn}

Solution:

Given, n(A) = m

n(B) = n

∴ n(A × B) = mn

Then, the number of relations from A to is 2^{mn}

20. If R is a relation on a finite set having n elements, then the number of relations on A is:

a. 2^n

b. 2^{n^2}

c. n^2

d. n^n

Ans. :

b. 2^{n^2}

Solution:

Given, A finite set with n elements

Its Cartesian product with itself will have n^2 elements.

∴ Number of relations on $A = 2^{n^2}$

21. If $e^{f(x)} = \frac{10+x}{10-x}$, $x \in (-10, 10)$ and $f(x) = kf\left(\frac{200x}{100+x^2}\right)$, then k =

a. 0.5

b. 0.6

c. 0.7

d. 0.8

Ans. :

a. 0.5

Solution:

$e^{f(x)} = \frac{10+x}{10-x}$

$\Rightarrow f(x) = \log_e\left(\frac{10+x}{10-x}\right) \ldots (i)$

$\Rightarrow f(x) = kf\left(\frac{200x}{100+x^2}\right)$

$\Rightarrow \log_e\left(\frac{10+x}{10-x}\right) = k\log_e\left(\frac{10+\frac{200x}{100+x^2}}{10-\frac{200x}{100+x^2}}\right)$ {from (1)}

$\Rightarrow \log_e\left(\frac{10+x}{10-x}\right) = k\log_e\left(\frac{1000+10x^2+200x}{1000+10x^2-200x}\right)$

$\Rightarrow \log_e\left(\frac{10+x}{10-x}\right) = k\log_e\left(\frac{(x+10)^2}{(x-10)^2}\right)$

$\Rightarrow \log_e\left(\frac{10+x}{10-x}\right) = 2k\log_e\frac{(x+10)}{(x+10)}$

$\Rightarrow 1 = 2k$

$\Rightarrow k = \frac{1}{2} = 0.5$

22. The domain of definition of the function $f(x) = \sqrt{\frac{x-2}{x+2}} + \sqrt{\frac{1-x}{1+x}}$ is:

a. $(-\infty, -2] \cap [2, -\infty)$

b. $[-1, 1]$

c. ϕ

d. None of these.

Ans. :

c. ϕ

Solution:

$f(x) = \sqrt{\frac{x-2}{x+2}} + \sqrt{\frac{1-x}{1+x}}$

For f(x) to be defined,

$x + 2 \neq 0$

$\Rightarrow x \neq -2 \ldots (i)$

And $1 + x \neq 0$

$\Rightarrow x \neq -1 \ldots (ii)$

Also, $\frac{x-2}{x+2} \geq 0$

$\Rightarrow \frac{(x-2)(x-2)}{(x-2)^2} \geq 0$

$\Rightarrow (x-2)(x+2) \geq 0$

$\Rightarrow x \in (\infty, -2) \cup [2, \infty) \ldots (iii)$

And $\frac{1-x}{1+x} \geq 0$

$\Rightarrow \frac{(1-x)(1+x)}{(1+x)^2} \geq 0$

$\Rightarrow (1-x)(1+x) \geq 0$

$\Rightarrow \ x \in (-\infty, -1) \cup [1, \infty) \ \ldots(\text{iv})$

From (i), (ii), (iii) and (iv) we get

$x \ \in \phi$

Thus, domain $(f(x)) = \phi$

23. The domain of definition of $f(x) = \sqrt{x-3-2\sqrt{x-4}} - \sqrt{x-3+2\sqrt{x-4}}$ is:

a. $[4, \infty)$

b. $(-\infty, 4]$

c. $(4, \infty)$

d. $(-\infty, 4)$

Ans. :

a. $[4, \infty)$

Solution:

$f(x) = \sqrt{x-3-2\sqrt{x-4}} - \sqrt{x-3+2\sqrt{x-4}}$

For f(x) to be defined, $x-4 \geq 0$

$\Rightarrow x-4 \geq 0$

$\Rightarrow x \geq 4 \ldots(\text{i})$

Also, $x-3-2\sqrt{x-4} \geq 0$

$\Rightarrow x-3-2\sqrt{x-4} \geq 0$

$\Rightarrow x-3 \geq 2\sqrt{x-4}$

$\Rightarrow (x-3)^2 \geq (2\sqrt{x-4})^2$

$\Rightarrow x^2+9-6x \geq 4(x-4)$

$\Rightarrow x^2-10x+25 \geq 0$

$\Rightarrow (x-5)^2 \geq 0,$ which is always true.

Similarly, $x-3+2\sqrt{x-4} \geq 0$ is always true.

Thus, domain $(f(x)) = [4, \infty)$

24. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ and $g(x) = \frac{3x+x^3}{1+3x^2}$, then f(g(x)) is equal to:

a. f(3x)

b. $\{f(x)\}^3$

c. 3f(x)

d. -f(x)

Ans. :

c. 3f(x)

Solution:

$f(x) = \log\left(\frac{1+x}{1-x}\right)$ and $g(x) = \frac{3x+x^3}{1+3x^2}$

Now,

$\frac{1+g(x)}{1-g(x)} = \frac{1+\frac{3x+x^3}{1+3x^2}}{1-\frac{3x+x^3}{1+3x^2}}$

$= \frac{1+3x^2+3x+x^3}{1+3x^2-3x-x^3}$

$= \frac{(1+x)^3}{(1-x)^3}$

Then, $f(g(x)) = \log = \log\left(\frac{1+g(x)}{1-g(x)}\right)$

$= \log\left(\frac{1+x}{1-x}\right)^3$

$= 3f(x)$

25. If $f(x) = \frac{2^x+2^{-x}}{2}$, then f(x + y)f(x - y) is equal to:

a. $\frac{1}{2}\left[f(2x)+f(2y)\right]$

b. $\frac{1}{2}\left[f(2x)-f(2y)\right]$

c. $\frac{1}{4}\left[f(2x)+f(2y)\right]$

d. $\frac{1}{4}\left[f(2x)-f(2y)\right]$

Ans. :

a. $\frac{1}{2}\left[f(2x)+f(2y)\right]$

Solution:

Given,

$f(x) = \frac{2^x+2^{-x}}{2}$

Now,

$f(x+y)f(x-y) = \left(\frac{2^{x+y}+2^{-x-y}}{2}\right)\left(\frac{2^{x-y}+2^{-x+y}}{2}\right)$

$\Rightarrow\ f(x+y)f(x-y) = \frac{1}{4}\left(2^{2x}+2^{-2y}+2^{2y}+2^{-2x}\right)$

$\Rightarrow\ f(x+y)f(x-y) = \frac{1}{2}\left(\frac{2^{2x}+2^{-2x}}{2}+\frac{2^{2y}+2^{-2y}}{2}\right)$

$\Rightarrow f(x+y)f(x-y) = \frac{1}{2}\left[f(2x)+f(2y)\right]$

26. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to:

a. $\{f(x)\}^2$

b. $\{f(x)\}^3$

c. 2f(x)

d. 3f(x)

Ans. :

c. 2f(x)

Solution:

$f(x) = \log\left(\frac{1+x}{1-x}\right)$

Then, $f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right)$

$= \log\left(\frac{\frac{1+x^2+2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right)$

$= \log\left(\frac{(1+x)^2}{(1-x)^2}\right)$

$= 2\log\left(\frac{1+x}{1-x}\right)$

$= 2(f(x))$

27. If $f(x) = \frac{\sin^4 x+\cos^2 x}{\sin^2 x+\cos^4 x}$ for $x \in R$, then f(2002) =

a. 1

b. 2

c. 3

d. 4

Ans. :

a. 1

Solution:

Given,

$f(x) = \frac{\sin^4 x+\cos^2 x}{\sin^2 x+\cos^4 x}$

On dividing the numerator and denominator by $\cos^4 x$, we get

$f(x) = \frac{\tan^4 x+\sec^2 x}{1+\tan^2 x \sec^2 x}$

$= \frac{1+\tan^4 x+\tan^2 x}{1+\tan^2 x(1+\tan^2 x)}$

$= \frac{1+\tan^4 x+\tan^2 x}{1+\tan^4 x+\tan^2 x} = 1 \text{ (For every } x \in R)$

$\text{For } x = 2002,$

We have,

$f(2002) = 1$

28. If f : R → R be given by for all $f(x) = \frac{4^x}{4^x+2}\ x \in R$, then:

a. f(x) = f(1 - x)

b. f(x) + f(1 - x) = 0

c. f(x) + f(1 - x) = 1

d. f(x) + f(x - 1) = 1

Ans. :

c. f(x) + f(1 - x) = 1

Solution:

$f(x) = \frac{4^x}{4^x+2}\ x \in R,$

$f(1-x) = \frac{4^{1-x}}{4^{1-x}+2}$

$= \frac{4}{2 \times 4^x + 4}$

$= \frac{2}{4^x + 2}$

$f(x) + f(1-x) = \frac{4^x}{4^x+2} + \frac{2}{4^x+2}$

$= \frac{4^x+2}{4^x+2} = 1$

29. The domain of the function f given by $f(x) = \frac{x^2+2x+1}{x^2-x-6}$.

a. R – {3, –2}

b. R – {–3, 2}

c. R – [3, –2]

d. R – (3, –2)

Ans. :

a. R – {3, –2}

Solution:

Given that: $f(x) = \frac{x^2+2x+1}{x^2-x-6}$

f(x) is defined if $x^2 - x - 6 \neq 0$

$\Rightarrow x^2 - 3x + 2x - 6 \neq 0$

$\Rightarrow (x-3)(x+2) \neq 0$

$\Rightarrow x \neq -2, x \neq 3$

So, the domain of f(x) = R - {-2, 3}

30. Domain of $\sqrt{a^2 - x^2}\,(a > 0)$ is.

a. (-a, a)

b. [-a, a]

c. [0, a]

d. (-a, 0]

Ans. :

b. [-a, a]

Solution:

We have $f(x)\sqrt{a^2 - x^2}$

Clearly f(x) is defined, if $a^2 - x^2 \geq 0$

$\Rightarrow x^2 \leq a^2$

$\Rightarrow -a \leq x \leq a\ [\because a > 0]$

$\therefore$ Domain of f is [-a, a]

31. If $A = \{a, b, c, d\}$ and $B = \{p, q, r, s\}$ then a realtion from A to B is :

(A) $\{(a,p),(b,r),(c,r)\}$

(B) $\{(a,p),(b,q),(c,r),(s,d)\}$

(C) $\{(b,a),(q,b),(c,r)\}$

(D) $\{(c,s),(d,s),(r,a),(q,b)\}$

Ans.: (A) $\{(a,p),(b,r),(c,r)\}$

32. A relation R is defined on N such that $xRy \Leftrightarrow x + 4y = 16$, then the range of R is :

(A) $\{1,2,4\}$ (B) $\{1,3,4\}$ (C) $\{1,2,3\}$ (D) $\{2,3,4\}$

Ans. : (C) $\{1,2,3\}$

33. The set builder form of relation $\{(1,2),(2,5),(3,10)$, $(4,17)$ .... } on N is :

(A) $\{(x,y)/x,y\in N, y=2x+1\}$ (B) $\left\{(x,y)/x,y\in N, y=x^2+1\right\}$

(C) $\{(x,y)/x,y\in N, y=3x-1\}$ (D) $\{(x,y)/x,y\in N, y=x+3\}$

Ans. : (B) $\left\{(x,y)/x,y\in N, y=x^2+1\right\}$

34. If R is a relation on $\{1,2,3\}$ such that x is divisor of y, then R is :

(A) $\{(1,1),(2,2),(3,3),(1,2),(1,3)\}$ (B) $\{(1,1),(2,2),(3,3)\}$

(C) $\{(1,1),(1,2),(1,3)\}$ (D) $\{(1,1),(1,2),(1,3),(2,3)\}$

Ans.: (A) $\{(1,1),(2,2),(3,3),(1,2),(1,3)\}$

35. If $f:R\to R$ is a function $f(x+y)=f(x)+f(y)\forall x$, $y\in R$ and $f(1)=7$ then $\sum_{r=1}^{n} f(r)$ is :

(A) $\frac{7n}{2}$ (B) $\frac{7(n+1)}{2}$ (C) $7n(n+1)$ (D) $\frac{7n(n+1)}{2}$

Ans. : (D) $\frac{7n(n+1)}{2}$

36. Which of the following rules is not a function from R to R ?

(A) $f(x)=x^2$ (B) $f(x)=\sqrt{x}$ (C) $f(x)=x^{1/3}$ (D) $f(x)=x^3$

Ans. : (B) $f(x)=\sqrt{x}$

37. If $A=\{2,4,5,7\}, B=\{2,3,4,6,8\}$ and a relation R is defined from set A to set B such that $xRy \Leftrightarrow x$ is divisor of y, then range of R is :

(A) $\{2,3,6,8\}$ (B) $\{2,4,6,7\}$ (C) $\{2,4,6,8\}$ (D) $\{4,6,8\}$

Ans. : (C) $\{2,4,6,8\}$

38. If $n(A)=p, n(B)=q$ then the number of relation from A to B is :

(A) $2^{pq}+1$ (B) $2^{pq}-1$ (C) 2^{pq-1} (D) 2^{pq}

Ans. : (B) $2^{pq}-1$

39. If $f:R\to R, f(x)=\sin \pi x$ then the range of f is :

(A) $\{x/-\pi\le x\le \pi\}$ (B) $\{x/-\pi<x<\pi\}$ (C) $\{x/-1<x<1\}$ (D) $\{x/-1\le x\le 1\}$

Ans. : (D) $\{x/-1\le x\le 1\}$

40. The range of function $f(x)=\cos\frac{x}{3}$ is :

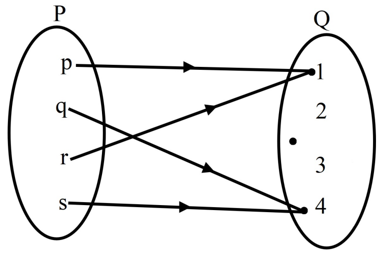
(A) $(0,\infty)$ (B) $(-\frac{1}{3},\frac{1}{3})$ (C) $[-1,1]$ (D) $[0,1]$

Ans. : (C) $[-1,1]$

*** A statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option.** **[3]**

41. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

Assertion: The following arrow diagram represents a function.



Reason: Let f : R - {2} → R be defined by $f(x) = \frac{x^2 - 4}{x - 2}$ and g : R → R be defined by g(x) = x + 3, Then, f = g.

a. A is true, R is true; R is a correct explanation of A.
b. A is true, R is true; R is not a correct explanation of A.
c. A is true; R is false.
d. A is false; R is true.

Ans. :

c. A is true; R is false.

42. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

Assertion: If f : R → R and g : R → R are defined by f(x) = 2x + 3 and g(x) = x^2 + 7, then the values of x such that g{f(x)} = 8 are -1 and 2. **Reason:** If f : R → R be given by $f(x) = \frac{4^x}{4^x + 2}$ for all $x \in R$, then f(x) + f(1 - x) = 1.

a. A is true, R is true; R is a correct explanation of A.
b. A is true, R is true; R is not a correct explanation of A.
c. A is true; R is false.
d. A is false; R is true.

Ans. :

d. A is false; R is true.

Solution:

Assertion: We have,

f (x) = 2x + 3, g(x) = x^2 + 7

g [f(x)] = 8

⇒ g(2x + 3) = 8

⇒ $(2x + 3)^2 + 7 = 8$

⇒$(2x + 3)^2 = 1$

$\Rightarrow 2x + 3 = \pm 1,$

2x + 3 = -1

or 2x + 3 = 1, then

⇒ x = -1, x = -2

Reason: Now, $f(x) = \frac{4^x}{4^x + 2}$

$f(1-x)=\frac{4^{1-x}}{4^{1-x}+2}$

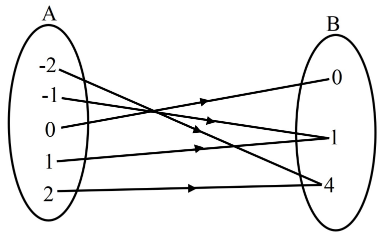
$\therefore f(x)+f(1-x)=\frac{4^{x}}{4^{x}+2}+\frac{4^{1-x}}{4^{1-x}+2}$

$=\frac{4^{x}}{4^{x}+2}+\frac{\frac{4}{4^{x}}}{4+2\cdot 4^{x}}$

$=\frac{4^{x}}{4^{x}+2}+\frac{2}{4^{x}+2}$

$=\frac{4^{x}+2}{4^{x}+2}=1.$

43. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
Consider the following statements



Assertion: The figure shows a relationship between the sets A and B. Then, the relation in Set - builder form is {(x, y) : y = x^2, x, $y \in N$ and $-2, \leq x \leq 2$}.
Reason: The above Relation in Roster form is {(-1, 1), (2, 4), (0, 0), (1, 1), (2, 4)}.

a. A is true, R is true; R is a correct explanation of A.
b. A is true, R is true; R is not a correct explanation of A.
c. A is true; R is false.
d. A is false; R is true.

Ans. :
d. A is false; R is true.
Solution:
Assertion: In Set - builder form,
R = {(x, y) : y = x^2, x, $y \in N$ and $-2, \leq x \leq 2$} $[\because 0 \in N]$
Reason: The relation shown in figure is represented in Roster form as
R = {(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)}
We observe that, second element of each ordered pair is the square of first element.

*** Answer the following questions in one sentence. [1 Marks Each]** **[17]**

44. If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$, find the values of x and y.

Ans. : Given, $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$
Comparing corresponding elements,
$\Rightarrow \frac{x}{3}+1=\frac{5}{3}$ and $y-\frac{2}{3}=\frac{1}{3}$
$\Rightarrow \quad \frac{x}{3}=\frac{5}{3}-1$ and $y=\frac{1}{3}+\frac{2}{3}$
$\Rightarrow \quad \frac{x}{3}=\frac{5-3}{3}$ and $y=\frac{3}{3}$

$\Rightarrow \quad \frac{x}{3}=\frac{2}{3}$ and $y=1$

$\therefore x=2$ and $y=1$

45. If the set A has 3 elements and set B = {3, 4, 5}, then find the number of elements in A $\times$ B.

Ans. : Set A has 3 elements and set B also has 3 elements,

$\therefore$ the number of elements in $A \times B = 3 \times 3 = 9$

46. If $A \times B$= {(a, x), (a, y), (b, x), (b, y)}, find A and B.

Ans. : Here $A \times B$ = {(a, x), (a, y), (b, x), (b, y)}

A = set of first elements = {a, b}

B = set of second elements = {x, y}

47. Let A and B be two sets such that n(A) = 3 and n(B) = 2. If (x, 1), (y, 2), (z, 1) are in A $\times$ B., find A and B, where x, y and z are distinct elements.

Ans. : Here (x, 1) $\in$ A $\times$ B $\Rightarrow$ x $\in$ A and 1 $\in$ B

(y, 2) $\in$ A $\times$ B $\Rightarrow$ y $\in$ A and 2 $\in$ B

(z, 1) $\in$ A $\times$ B $\Rightarrow$ z $\in$ A and 1

It is given that n(A) = 3 and n(B) = 2

$\therefore$ A = {x, y, z}

and B = {1, 2}

48. Let A = {1, 2, 3, . . . 14}. Define a relation R from A to A by R = {(x, y): 3x - y = 0, where $x, y \in A$}. Write down its domain, codomain and range.

Ans. : Here A = {1, 2, 3, . . . , 14}

We shall consider the ordered pairs which satisfy 3x - y = 0

They are (1, 3), (2, 6), (3, 9) and (4, 12)

Thus R = {(1, 3), (2, 6), (3, 9), (4, 12)}

$\therefore$ Domain = {1, 2, 3, 4}

Range = {3, 6, 9, 12}

Codomain = {1, 2,3, . . . , 14}

49. If A = {9, 10, 11, 12,13} and f : A $\rightarrow$ N be defined by f(n) = the highest prime factor of n, then find the range of f.

Ans. : f(n) = Highest prime factor of n

$\therefore f(9)=3, f(10)=5$

$f(11)=11, f(12)=3$ and $f(13)=13$

$\Rightarrow$ f = $\{(9,3),(10,5),(11,11),(12,3),(13,13)$ }

Hence, range of f = {3, 5,11,13}.

50. If (x + 1, y – 2) = (3, 1), find the values of x and y

Ans. : Since the ordered pairs are equal, the corresponding elements are equal.Then,we have,

$x + 1 = 3$ and $y - 2 = 1$

Solving we get $x = 2$ and $y = 3$

51. If R is a relation defined on the set Z of integers by the rule $(x, y) \in R \Leftrightarrow x^2 + y^2 = 9,$ then write domain of R.

Ans. : We have,

$(x, y) \in R \Leftrightarrow x^2 + y^2 = 9$

$\Rightarrow y^2 = 9 - x^2$

$\Rightarrow y = \sqrt{9 - x^2}$

Putting x = -3, 0, 3, we get $y = 0, \pm 3, 0$ respectively.

For all other values of x, we get $y \notin z$

$\therefore \ \text{Domain}(R) = \{-3, 0, 3\}$

52. If A = {1, 2, 3}, B = {4, 5, 6}, the given following are relations from A to B? Give reason in support of your answer.

{(4, 2), (4, 3), (5, 1)}

Ans. : We have,

A = {1, 2, 3} and B = {4, 5, 6}

{(4, 2), (4, 3), (5, 1)} is not a relation from A to B as it is not a subset of A × B.

53. What is the fundamental difference between a relation and a function? Is every relation a function?

Ans. : Function is a type of relation. But in a function no two ordered pairs have the same first element. For eg. R_1 and R_2 are two relations.

Clearly, R_1 is a function, but R_2 is not a function because two ordered pairs (1, 2) and (1, 4) have the same first element.

This means every function is a relation but every relation is not a function.

54. Write the domain and range of function f(x) given by $f(x) = \frac{1}{\sqrt{x - |x|}}$

Ans. : We have,

$f(x) = \frac{1}{\sqrt{x - |x|}}$

We know that,

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

$$\Rightarrow x - |x| = |x| = \begin{cases} x - x = 0, & \text{if } x \geq 0 \\ x + x = 2x, & \text{if } x < 0 \end{cases}$$

$\Rightarrow x - |x| \leq 0$ for all x

$\Rightarrow \frac{1}{\sqrt{x - |x|}}$ does not take real values for any $x \in R$

$\Rightarrow f(x)$ is not defined for any $x \in R$

Hence, domain $(f) = \phi = \text{Range}(f)$

55. Find the domain of each of the following functions given by:

$f(x) = \frac{3x}{28-x}$

Ans. : $f(x) = \frac{3x}{28-x}$

Clearly, f(x) is not defined, if 28 - x = 0

$\Rightarrow x \neq 28$

$\therefore$ Domain of f = R -{28}

56. If f and g are real function defined by f(x) = x^2 + 7 and g(x) = 3x + 5, find following:

f(3) + g (– 5)

Ans. : Given that: f(x) = x^2 + 7 and g(x) = 3x + 5

f(3) + g(– 5) = [$(3)^2$ + 7] + [3(-5) + 5]

= (9 + 7) + (-15 + 5) = 16 -10 = 6

Hence, f(3) + g(-5) = 6.

57. A relation R is defined on a set $A = \{1,2,3,4,5,6\}$ such that $xRy \Leftrightarrow x + 2y = 8$, then write the domain of R.

Ans. :

$\text{Given,} \quad x + 2y = 8$

$\therefore \quad 2y = 8 - x$

$\Rightarrow \quad y = \frac{8-x}{2}$

2 R 3, 4 R 2, 6 R 1

$\therefore$ Domain of $R = \{2,4,6\}$

58. If $f(x) = \frac{x}{x+1}$, then write the value of $f\left(\frac{p}{q}\right)$.

Ans. : $f\left(\frac{p}{q}\right) = \frac{p/q}{\frac{p}{q}+1} = \frac{p/q}{\frac{p+q}{q}} = \frac{p}{p+q}$

59. If $A = \{2,3,5,7\}$ and $f : A \to N, f(x) = x^3 + 2$, then find the range of function.

Ans. :

$f(2) = 2^3 + 2 = 8 + 2 = 10$

$f(3) = 3^3 + 2 = 27 + 2 = 29$

$f(5) = 5^3 + 2 = 125 + 2 = 127$

$f(7) = 7^3 + 2 = 343 + 2 = 345$

Hence, range of function $= \{10,29,127,345\}$

60. If $f(x) = \tan x$ then write the value of $f(x) + f(\pi - x)$.

Ans. :

$f(x) = \tan x, f(\pi - x) = \tan(\pi - x) = -\tan x$

$\therefore f(x) + f(\pi - x) = \tan x - \tan x = 0$

*** Given section consists of questions of 2 marks each.** **[26]**

61. Let $f=\left\{\left(x, \frac{x^2}{1+x^2}\right) : x \in R\right\}$ be a function from R into R. Determine the range of f.

Ans. : Here $f(x)=\frac{x^2}{1+x^2}$

Put $y=\frac{x^2}{1+x^2}$ $\Rightarrow$ $y + yx^2 = x^2$ $\Rightarrow$ $x^2(1 - y) = y$

$\Rightarrow x^2=\frac{y}{1-y} \Rightarrow x=\pm\sqrt{\frac{y}{1-y}}$

$\frac{y}{1-y} \geq 0$

$\Rightarrow \frac{y}{y-1} \leq 0$

$\Rightarrow 0 \leq \mathrm{y}<1$

$\Rightarrow \mathrm{y} \in[0,1)$

$\therefore$Range of f(x) = [0, 1)

62. Find the domain of the function $f(x)=\frac{x^2+2x+1}{x^2-8x+12}$.

Ans. : Here $f(x)=\frac{x^2+2x+1}{x^2-8x+12}$

f (x) is a rational function of x.

f (x) assumes real values of all x except for those values of x for which

$x^2 - 8x + 12 = 0$

$\Rightarrow$ (x - 6)(x - 2) = 0

$\Rightarrow$ x = 2, 6

$\therefore$ Domain of function = R - {2, 6}.

63. If f (x) = x^2, find $\frac{f(1.1)-f(1)}{(1.1-1)}$

Ans. : Here f (x) = x^2

At x = 1.1

f(1.1) = $(1.1)^2$ = 1.21

f(1)=$(1)^2$ = 1

$\therefore \frac{f(1.1)-f(1)}{(1.1-1)}=\frac{1.21-1}{0.1}=\frac{0.21}{0.1}=2.1$

64. Find the inverse relation R^{-1} in the following case:

$\mathrm{R}=\{(\mathrm{x}, \mathrm{y}) : \mathrm{x},\ \mathrm{y} \in \mathrm{N},\ \mathrm{x}+2\mathrm{y}=8\}$

Ans. : We have,

$\mathrm{R}=\{(\mathrm{x}, \mathrm{y}) : \mathrm{x},\ \mathrm{y} \in \mathrm{N},\ \mathrm{x}+2\mathrm{y}=8\}$

Now,

x + 2y = 8

⇒ x = 8 - 2y

Putting y = 1, 2, 3 we get x = 6, 4, 2 respectively.

For y = 4, we get $\mathrm{x}=0 \notin \mathrm{N}$ Also for $\mathrm{y}>4,\ \mathrm{x} \notin \mathrm{N}$

$\therefore\ \mathrm{R}=\{(6,1),(4,2),(2,3)\}$

Thus,

$\mathrm{R}^{-1}=\{(1,6),(2,4),(3,2)\}$

$\Rightarrow \mathrm{R}^{-1}=\{(3,2),(2,4),(1,6)\}$

65. If $\mathrm{a}\in\{2,4,6,9\}$ and $\mathrm{b}\in\{4,6,18,27\}$, then form the set of all ordered pairs (a, b) such that a divides b and a < b.

Ans. : We have,

$\mathrm{a}\in\{2,4,6,9\}$

and, $\mathrm{b}\in\{4,6,18,27\}$

Now, $\frac{\mathrm{a}}{\mathrm{b}}$ stands for 'a divides b'. For the elements of the given sets, we find that $\frac{2}{4},\frac{2}{6},\frac{2}{18},\frac{6}{18},\frac{9}{18}$ and $\frac{9}{27}$

∴ {(2, 4), (2, 6), (2, 18), (6, 18), (9, 18), (9, 27)} are the required set of ordered pairs (a, b).

66. Find the inverse relation R^{-1} in the following case:

R is a relation from {11, 12, 13} to (8, 10, 12} defined by y = x - 3.

Ans. : We have,

R is a relation from {11, 12, 13} to (8, 10, 12} defined by y = x - 3.

Now,

y = x - 3

Putting x = 11, 12, 13 we gets y = 8, 9, 10 respectively.

$\Rightarrow (11,8)\in\mathrm{R},(12,9)\notin\mathrm{R}$

Thus,

$\mathrm{R}=\{(11,8),(13,10)\}$

$\Rightarrow \mathrm{R}^{-1}=\{(8,11),(10,13)\}$

67. If f : R → R be defined by $f(x) = x^2 + 1$, then find $f^{-1}\{17\}$ and $f^{-1}\{-3\}$.

Ans. : We know that,

if f : A → 13

such that $\mathrm{y}\in 3$. Then,

$\mathrm{f}^{-1}(\mathrm{y})=\{\mathrm{x}\in\mathrm{A}:\mathrm{f}(\mathrm{x})=\mathrm{y}\}$ In other words, $f^{-1}(y)$ is the set of pre-images of y.

Let $f^{-1}\{17\}$ = x. Then, f(x) = 17

⇒ $x^2 + 1 = 17$

⇒ $x^2 = 17 - 1 = 16$

$\Rightarrow\ \mathrm{x}=\pm 4$

Let $f^{-1}\{-3\}$ = x. Then, f(x) = -3

⇒ $x^2 + 1 = -3$

⇒ $x^2 = -3 - 1 = -4$

$\Rightarrow x = \sqrt{-4}$

$\therefore\ f^{-1}\{-3\} = \theta$

68. Find the domain of the following real valued functions of real variable:

$f(x) = \frac{x^2+2x+1}{x^2-8x+12}$

Ans. : Given,

$f(x) = \frac{x^2+2x+1}{x^2-8x+12}$

$= \frac{x^2+2x+1}{x^2-6x-2x+12}$

$= \frac{x^2+2x+1}{x(x-6)-2(x-6)}$

$= \frac{x^2+2x+1}{(x-6)(x-2)}$

Domain of f, Clearly, f (x) is a rational function of x as $\frac{x^2+2x+1}{x^2-8x+12}$ is a rational expression.

Clearly, f(x) assumes real values for all x except for all those values of x for which x^2 - 8x + 12 = 0, i.e. x = 2, 6.

Hence, domain (f) = R - {2, 6}

69. If $f(x) = \frac{x-1}{x+1}$, then show that.

$f\left(\frac{1}{x}\right) = -f(x)$

Ans. : Given that: $f(x) = \frac{x-1}{x+1}$

$f\left(\frac{1}{x}\right) = \frac{\frac{1}{x}-1}{\frac{1}{x}+1} = \frac{1-x}{1+x} = \frac{-(x-1)}{x+1} = -f(x)$

Hence, $f\left(\frac{1}{x}\right) = -f(x)$

70. If f and g are real function defined by f(x) = x^2 + 7 and g(x) = 3x + 5, find following:

$\frac{f(t)-f(5)}{t-5}$, if $t \neq 5$

Ans. : $\frac{f(t)-f(5)}{t-5}, t \neq 5 = \frac{(t^2+7)-((5)^2 7)}{t-5} = \frac{(t^2+7)-(5^2+7)}{t-5}$

$= \frac{t^2+7-32}{t-5} = \frac{t^2-25}{t-25} = t+5$

$\frac{t^2+7-32}{t-5} = \frac{t^2-25}{t-5} = \frac{(t-5)(t+5)}{t-5} = t+5$

Hence, $\frac{f(t)-f(5)}{t-5}, t \neq 5 = t+5.$

71. Find the domain of following function given by:

$f(x) = \frac{x^3-x+3}{x^2-1}$

Ans. : We have, $f(x) = \frac{x^3-x+3}{x^2-1}$

f(x) is not defined, if x^2 - 1 = 0

⇒ (x - 1)(x + 1) = 0

⇒ x = -1, 1

∴ Domain of f = R - {-1, 1}

72. Draw the graph of function $f(x) = |x| + 1$.

Ans. : Given, $f(x) = |x| + 1$

x	-2	-1	0	1	2
f(x)	3	2	1	2	3

Image

73. A relation R is defined on the set of integers such that $xRy \Leftrightarrow x^2 + y^2 = 25$ then write R and R^{-1} as the set of ordered pairs and also find its domain.

Ans. : Given relation R is defined as-

$$xRy \Leftrightarrow x^2 + y^2 = 25$$
$$\Rightarrow \quad y = \pm\sqrt{25 - x^2}$$

Here, $x = 0, \Rightarrow y = \pm 5 \therefore (0,5) \in R$ and $(0,-5) \in R$

$x = \pm 3 \Rightarrow y = \pm 4$

$$\therefore \quad (3,4) \in R, (-3,4) \in R, (3,-4) \in R \text{ and } -3,,4) \in R$$
$$x = \pm 4 \Rightarrow y = \pm 3$$
$$\therefore \quad (4,3) \in R, (4,-3) \in R, (-4,3) \in R \text{ and } (-4,-3) \in R$$
$$\text{and} \quad x = \pm 5 \Rightarrow y = 0$$
$$\therefore \quad (5,0) \in R \text{ and } (-5,0) \in R$$

We see here that for any other integer value of x, there is no integer value of y, which satisfies the given relation.

$$\text{So,} \quad R = \{(0,5), (0,-5), (3,4), (-3,4), (3,-4), (-3,-4), (4,3), (4,-3), (-4,3), (-4,-3), (5,0), (-5,0)\}$$
$$R^{-1} = \{(5,0), (-5,0), (4,3), (4,-3), (-4,3), (-4,-3), (3,4), (3,-4), (-3,4), (-3,-4), (0,5), (0,-5)\}$$

$$\text{Domain of } R = \{0, 3, -3, 4, -4, 5, -5\}$$
$$= \text{ Domain of } R^{-1}$$

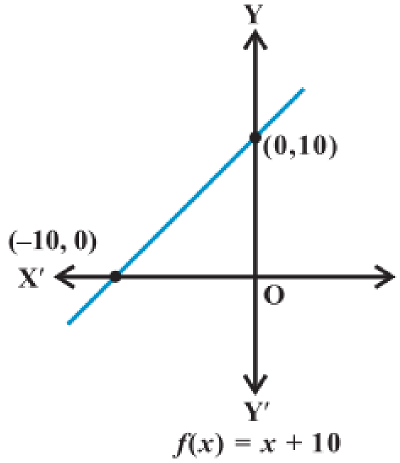
*** Given section consists of questions of 3 marks each.** **[66]**

74. Let R be the set of real numbers. Define the real function f: R → R by f(x) = x + 10 and sketch the graph of this function.

Ans. : Here we have f(x) = x + 10

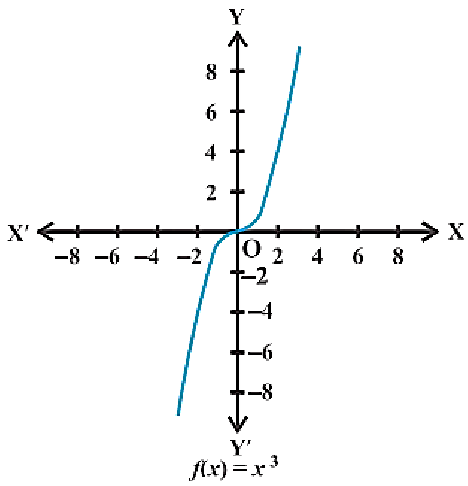
Here f(0) = 10, f(1) = 11, f(2) = 12, ..., f(10) = 20, etc., and f(−1) = 9, f(−2) = 8, ..., f(−10) = 0 and so on.

Therefore, the shape of the graph of the given function assumes the form as shown in figure below:



75. Draw the graph of the function f : R $\rightarrow$ R defined by f (x) = x^3, x $\in$ R.

Ans. : Given function is, f (x) = x^3, x $\in$ R.
We have f(0) = 0, f(1) = 1, f(–1) = –1, f(2) = 8, f(–2) = –8, f(3) = 27; f(–3) = –27, etc.
Therefore, f = {(x, x^3): x $\in$ R}. The graph of f is given in figure showjbelow:



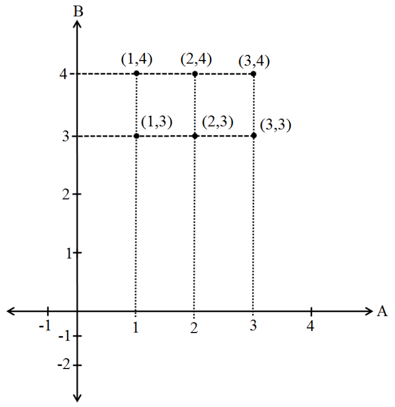
76. Let f, g: $R \rightarrow R$ be defined, respectively by f(x) = x + 1, g(x) = 2x – 3. Find f + g, f – g and $\frac{f}{g}$.

Ans. : Here f (x) = x + 1 and g (x) = 2x – 3
Now (f + g) (x) =f (x) + g(x) = x + 1 + 2x - 3 = 3x - 2
(f - g) (x) = f(x) - g(x) = x + 1- (2x - 3) = x + 1 - 2x + 3 = -x + 4
$\frac{(f)}{(g)}(x) = \frac{f(x)}{g(x)} = \frac{x+1}{2x-3}, x \neq \frac{3}{2}$

77. Let A = {1, 2, 3} and B = {3, 4}. Find A × B and show it graphically.

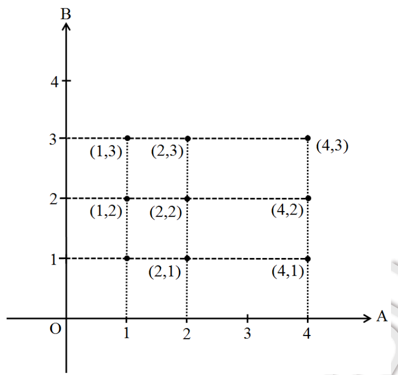
Ans. : We have,
A = {1, 2, 3} and B = {3, 4}
$\therefore$ A × B = {1, 2, 3} × {3, 4}
= {(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)}
To represent A × B graphically, follow the given steps:

Draw two mutually perpendicular lines-one horizontal and one vertical.
On the horizontal line, represent the elements of set A and on the vertical line, represent the elements of set B.
Draw vertical dotted lines through points representing elements of set A on the horizontal line and horizontal lines through points representing elements of set B on the vertical line.
The points of intersection of these lines will represent A × B graphically.



78. If A = {1, 2, 4} and B = {1, 2, 3}, represent following sets graphically:
A × B

Ans. : We have,
A = {1, 2, 4} and B = {1, 2, 3}
∴ A × B = {1, 2, 4} × {1, 2, 3}
= {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (4, 1), (4, 2), (4, 3)}
Hence, we represent A on the horizontal line and B on vertical line.
Graphical representation of A × B is as shown below:



79. If A = {1, 2, 3}, B = {3, 4} and C = {4, 5, 6}, find
$A \times (B \cap C)$

Ans. : We have,
$A = \{1,2,3\}$, $B = \{3,4\}$ and $C = \{4,5,6\}$
$\therefore\ B \cap C = \{3,4\} \cap \{4,5,6\} = \{4\}$
$\therefore\ A \times (B \cap C) = \{1,2,3\} \times \{4\}$
$= \{(1,4),(2,4),(3,4)\}$
$\Rightarrow A \times (B \cap C) = \{(1,4),(2,4),(3,4)\}$

80. i. If $\left(\frac{a}{3}+1, b-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$, find the values of a and b.

ii. f(x + 1, 1) = (3, y - 2), find the values of x and y.

Ans. :

i. By the definition of equality of ordered pairs,

$\left(\frac{a}{3}+1, b-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$

$\Rightarrow \frac{a}{3}+1=\frac{5}{3}$ and $b-\frac{2}{3}=\frac{1}{3}$

$\Rightarrow \frac{a}{3}=\frac{5}{3}=-1$ and $b=\frac{1}{3}+\frac{2}{3}$

$\Rightarrow \frac{a}{3}=\frac{5-3}{3}$ and $b=\frac{1+2}{3}$

$\Rightarrow \frac{a}{3}=\frac{2}{3}$ and $b=\frac{3}{3}$

$\Rightarrow a=2$ and $b=1$

ii. By the definition of equality of ordered pairs,

(x + 1, 1) = (3, y - 2)

⇒ x + 1 = 3 and 1 = y - 2

⇒ x = 3 - 1 and x + 2 = y

⇒ x = 2 and 3 = y

⇒ x = 2 and y = 3

81. If $a \in [-1,2,3,4]$ and $b \in [0,3,6]$, write the set of all ordered pairs (a, b) such that a + b= 5.

Ans. : We have,

a + b = 5

⇒ a = 5 - b

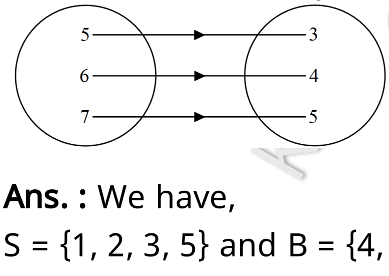
∴ b = 0 ⇒ a = 5 - 0 = 5,

b = 3 ⇒ a = 5 - 3 = 2,

b = 6 ⇒ a = 5 - 6 = -1

Hence, the required set of ordered pairs (a, b) is {(-1, 6), (2, 3), (5, 0)}

82. A = {1, 2, 3, 5} and B = {4, 6, 9}. Define a relation R from A to B by R = {(x, y): the difference between x and y is odd, $x \in A, y \in B\}$ Write R in Roster form.



Ans. : We have,

S = {1, 2, 3, 5} and B = {4, 6, 9}

It is given that,

R = {(x, y): the difference between x and y is odd, $x \in A, y \in B\}$

For the elements of the given sets A and B, we find that

$(1,4) \in \mathrm{R}, (1,6) \in \mathrm{R}, (2,9) \in \mathrm{R}, (3,4) \in \mathrm{R}, (3,6) \in \mathrm{R}, (5,4) \in \mathrm{R}$ and $(5,6)$
$\in \mathrm{R}$
$\therefore\ \mathrm{R} = \{(1,4),(1,6),(2,9),(3,4),(3,6),(5,4),(5,6)\}$
Hence, relation R in roster form is {(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)}

83. Let R be a relation on N × N defined by:
$(\mathrm{a}, \mathrm{b})\ \mathrm{R}\ (\mathrm{c}, \mathrm{d}) \Leftrightarrow \mathrm{a}+\mathrm{d}=\mathrm{b}+\mathrm{c}$ for all $(\mathrm{a}, \mathrm{b}),(\mathrm{c}, \mathrm{d}) \in \mathrm{N} \times \mathrm{N}$ Show that: $(\mathrm{a}, \mathrm{b})\ \mathrm{R}\ (\mathrm{c}, \mathrm{d})$ and $(\mathrm{c}, \mathrm{d})\ \mathrm{R}\ (\mathrm{e}, \mathrm{f}) \Rightarrow (\mathrm{a}, \mathrm{b})\ \mathrm{R}\ (\mathrm{e}, \mathrm{f})$ for all $(\mathrm{a}, \mathrm{b}),(\mathrm{c}, \mathrm{d}),(\mathrm{e}, \mathrm{f}) \in \mathrm{N} \times \mathrm{N}$

Ans. : We have,
$(\mathrm{a}, \mathrm{b})\ \mathrm{R}\ (\mathrm{c}, \mathrm{d}) \Leftrightarrow \mathrm{a}+\mathrm{d}=\mathrm{b}+\mathrm{c}$ for all $(\mathrm{a}, \mathrm{b}),(\mathrm{c}, \mathrm{d}) \in \mathrm{N} \times \mathrm{N}$
Now,
(a, b) R (c, d) and (c, d) R (e, f)
⇒ a + d = b + c and c + f = d + e
⇒ a + d + c + f = b + c + d + e [Adding]
⇒ a + f = b + e
⇒ (a, b) R (e, f)

84. If A = {1, 2, 3}, B = {3, 4} and C = {4, 5, 6}, find
$(\mathrm{A} \times \mathrm{B}) \cup (\mathrm{A} \cup \mathrm{C})$

Ans. : $(\mathrm{A} \times \mathrm{B}) \cup (\mathrm{A} \cup \mathrm{C})$
Now,
$(\mathrm{A} \times \mathrm{B}) = \{(1,3),(1,4),(2,3),(2,4),(3,3),(3,4)\}$
And,
$(\mathrm{A} \times \mathrm{C}) = \{(1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6)\}$
$\therefore\ (\mathrm{A} \times \mathrm{B}) \cup (\mathrm{A} \cup \mathrm{C})$
$= (1,3),(1,4),(1,5),(1,6),(2,3),(2,4),(2,5),(2,6),(3,3),(3,4),(3,5),(3,6)$

85. If A = {1, 2, 3}, B = {4}, C = {5}, then verify that:
$\mathrm{A} \times (\mathrm{B} \cap \mathrm{C}) = (\mathrm{A} \times \mathrm{B}) \cap (\mathrm{A} \times \mathrm{C})$

Ans. : We have,
$\mathrm{A} = \{1,2,3\},\ \mathrm{B} = \{4\}$ and $\mathrm{C} = \{5\}$
$\therefore\ \mathrm{B} \cap \mathrm{C} = \{4\} \cap \{5\} = \phi$
$\therefore\ \mathrm{A} \times (\mathrm{B} \cap \mathrm{C}) = \{1,2,3\} \times \phi$
$\Rightarrow \mathrm{A} \times (\mathrm{B} \cap \mathrm{C}) = \phi \ldots \text{(i)}$
Now,
$\mathrm{A} \times \mathrm{B} = \{1,2,3\} \times \{4\}$
$= \{(1,4),(2,4),(3,4)\}$
and, $\mathrm{A} \times \mathrm{C} = \{1,2,3\} \times \{5\}$
$= \{(1,5),(2,5),(3,5)\}$
$\therefore\ (\mathrm{A} \times \mathrm{B}) \cap (\mathrm{A} \times \mathrm{C}) = \{(1,4),(2,4),(3,4)\} \cup \{(1,5),(2,5),(3,5)\}$
$\Rightarrow (\mathrm{A} \times \mathrm{B}) \cup (\mathrm{A} \times \mathrm{C}) = \phi \ldots \text{(ii)}$
From equation (i) and (ii), we get

$A \times (B \cap C) = (A \times B) \cap (A \times C)$
Hence verified.

86. Let A and B be two sets. Show that the sets A × B and B × A have elements in common iff the sets A and B have an elements in common.

Ans. : Let (a, b) be an arbitrary element of $(A \times B) \cap (B \times A)$. Then,
$(a, b) \in (A \times B) \cap (B \times A)$
$\Leftrightarrow (a, b) \in A \times B$ and $(a, b) \in B \times A$
$\Leftrightarrow (a \in A \text{ and } b \in B)$ and $(a \in B \text{ and } b \in A)$
$\Leftrightarrow (a \in A \text{ and } a \in B)$ and $(b \in A \text{ and } b \in B)$
$\Leftrightarrow a \in A \cap B$ and $b \in A \cap B$
Hence, the sets A × B and B × A have an element in comon iff the sets A and B have an element in common.

87. Write the following relation as the sets of ordered pairs:
A relation R on the set {1, 2, 3, 4, 5, 6, 7}defined by $(x, y) \in R \Leftrightarrow x$ is relatively prime to y.

Ans. : We have,
It is given that relation R on the set {1, 2, 3, 4, 5, 6, 7} defined by $(x, y) \in R \Leftrightarrow x$ is relatively.
$\therefore\ (2,3) \in R, (2,5) \in R, (2,7) \in (3,2) \in R, (3,4) \in R, (3,5) \in R, (3,7) \in R,$
$(4,3) \in R, (4,5) \in R, (4,7) \in R, (5,2) \in R, (5,3) \in R, (5,4) \in R, (5,6)$
$\in R,$
$(5,7) \in R, (6,5) \in R, (6,7) \in R, (7,2) \in R, (7,3 \in R, (7,4)) \in R, (7,5)$
$\in R \text{ and } (7,6) \in R$

Thus,
$$R = \{(2,3), (2,5), (2,7), (3,2), (3,4), (3,5), (3,7), (4,3), (4,5), (4,7), (5,2),$$
$$(5,3), (5,4), (5,6), (5,7), (6,5), (6,7), (7,2), (7,3), (7,4), (7,5), (7,6)\}$$

88. If A = {1, 2, 3} and B = {2, 4}, what are A × B, B × A, A × A, B × B and $(A \times B) \cap (B \times A)$?

Ans. : We have,
A = {1, 2, 3} and B = {2, 4}
∴ A × B = {1, 2, 3} × {2, 4}
= {(1, 2), (1, 4), (2, 2), (2, 4), (3, 2), (3, 4)}
B × A = {2, 4} × {1, 2, 3}
= {(2, 1), (2, 2), (2, 3), (4, 1), (4, 2), (4, 3)}
A × A = {1, 2, 3} × {1, 2, 3}
= {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}
B × B = {2, 4} × {2, 4}
= {(2, 2), (2, 4), (4, 2), (4, 4)}

$(A \times B) \cap (B \times A)$
$= \{(1,2),(1,4),(2,2),(2,4),(3,2),(3,4)\}$
$\cap \{(2,1),(2,2),(2,3),(4,1),(4,2),(4,3)\}$
$= \{(2,2)\}$
$(A \times B) \cap (B \times A) = \{(2,2)\}$

89. Determine the domain and range of the relation R defined by:
R = {(x, x^3): x is a prime number less than 10}

Ans. : We have,
R = {(x, x^3): x is a prime number less than 10}
For the elements of the given sets, we find that
x = 2, 3, 5, 7
$\therefore\ (2,8) \in R, (3, 27) \in R, (5,127) \in R$ and $(7,343) \in R$
⇒ R = {(2, 8), (3, 27), (5, 125), (7, 343)}
Clearly, Domain (R) = {2, 3, 5, 7} and Range (R) = {8, 27, 125, 343}

90. The function f is defined by $f(x) = \begin{cases} x^2, & 0 \le x \le 3 \\ 3x, & 3 \le x \le 10 \end{cases}$

The relation g is defined by $g(x) = \begin{cases} x^2, & 0 \le x \le 2 \\ 3x, & 2 \le x \le 10 \end{cases}$ Show that f is a function and g is not a function.

Ans. : We have,
$f(x) = \begin{cases} x^2, & 0 \le x \le 3 \\ 3x, & 3 \le x \le 10 \end{cases}$
and $g(x) = \begin{cases} x^2, & 0 \le x \le 2 \\ 3x, & 2 \le x \le 10 \end{cases}$
Now, f(3) = $(3)^2$ = 9 and f(3) = 3 × 3 = 9
and g(2) = $(2)^2$ = 4 and g(2) = 3 × 2 = 6
We observe f(x) takes unique value at each point in its domain [0, 10]. However g(x) does not takes unique value at each in its domain [0, 10]
Hence, g(x) is not a function.

91. Let f and g be two real functions defined by $f(x) = \sqrt{x+1}$ and $g(x) = \sqrt{9 - x^2}$ Then describe the following functions:
f + g

Ans. : We have,
$f(x) = \sqrt{x+1}$ and $g(x) = \sqrt{9 - x^2}$
We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \ge -1$
So, domain $f = [-1, \infty]$
Clearly, $g(x) = \sqrt{9 - x^2}$ is defined for
$9 - x^2 \ge 0$

$\Rightarrow x^2 - 9 \leq 0$

$\Rightarrow x^2 - 3^2 \leq 0$

$\Rightarrow x \, \epsilon [-3, 3]$

$\therefore$ domain(g) $= [-3, 3]$

Now,

domain(f) $\cap$ domain(g)

$= [-1, \infty] \cap [-3, 3]$

$= [-1, 3]$

f + g : [-1, 3] → R is given by (f + g)(x) = f(x) + g(x)

$= \sqrt{x+1} + \sqrt{9 - x^2}$

92. Let X = {1, 2, 3, 4} and Y = {1, 5, 9, 11, 15, 16}. Determine which of the following sets are functions from X to Y:

 f_2 = {(1, 1), (2, 7), (3, 5)}

Ans. : We have,

f_2 = {(1, 1), (2, 7), (3, 5)}

f_2 is not a function from X to Y. because there is an element $4 \in x$ which is not. associated to any element of Y.

93. Find the domain of the following real valued functions of real variable:

 $f(x) = \frac{1}{\sqrt{x^2 - 1}}$

Ans. : Given,

$f(x) = \frac{1}{\sqrt{x^2 - 1}}$

Clearly, f(x) is defined for $x^2 - 1 > 0$

$(x+1)(x-1) > 0$ [Since $a^2 - b^2 = (a + b)(a - b)$]

$x < -1$ and $x > 1$

$x \, \epsilon (-\infty, -1) \cup (1, \infty)$

Hence, domain $(f) = (-\infty, -1) \cup (1, \infty)$

94. Find the range of the following function given by:

 $f(x) = \frac{3}{2 - x^2}$

Ans. : We have, $f(x) = \frac{3}{2 - x^2} = y$ (let)

$\Rightarrow 2 - x^2 = \frac{3}{y}$

$\Rightarrow x^2 = 2 - \frac{3}{y}$

Since $x^2 \geq 0, 2 - \frac{3}{y} \geq 0$

$\Rightarrow \frac{2y - 3}{y} \geq 0$

$\Rightarrow 2y - 3 \geq 0 y > 0 \Rightarrow 2y - 3 \leq 0 y < 0$

$\Rightarrow y \geq \frac{3}{2} y < 0$

$\Rightarrow y \in (-\infty, 0) \cup [\frac{3}{2}, \infty)$

$\therefore$ Range of f is $(-\infty, 0) \cup [\frac{3}{2}, \infty)$

95. If $f(x) = y = \frac{ax-b}{cx-a}$ then prove that f(y) = x.

Ans. : We have, $f(x) = y = \frac{ax-b}{cx-a}$

$\therefore f(y) = \frac{ay-b}{cy-a} = \frac{a\left(\frac{ax-b}{cx-a}\right)-b}{c\left(\frac{ax-b}{cx-a}\right)-a}$

$= \frac{a(ax-b)-b(cx-a)}{c(ax-b)-a(cx-a)}$

$= \frac{a^2x-ab-bcx+ab}{acx-bc-acx+a^2}$

$= \frac{a^2x-bcx}{a^2-bc}$

$= \frac{x(a^2-bc)}{(a^2-bc)}$

$\therefore f(y) = x$

*** Given section consists of questions of 5 marks each.** **[60]**

96. If A = {1, 2, 3}, B = {4}, C = {5}, then verify that:

$A \times (B \cup C) = (A \times B) \cup (A \times C)$

Ans. : We have,

$A = \{1, 2, 3\} \times \{4\}$

$\therefore\ B \cup C = \{4\} \cup \{5\} = \{4, 5\}$

$\therefore\ A \times (B \cup C) = \{1, 2, 3\} \times \{4, 5\}$

$\Rightarrow A \times (B \cup C) = \{(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)\} \ldots (i)$

Now,

$A \times B = \{1, 2, 3\} \times \{4\}$

$= \{(1,4), (2,4), (3,4)\}$

and, $A \times C = \{1, 2, 3\} \times \{5\}$

$= \{(1,5), (2,5), (3,5)\}$

$\therefore\ (A \times B) \cup (A \times C) = \{(1,4), (2,4), (3,4)\} \cup \{(1,5), (2,5), (3,5)\}$

$\Rightarrow (A \times B) \cup (A \times C) = \{(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)\} \ldots (ii)$

From equation (i) and (ii), we get

$A \times (B \cup C) = (A \times B) \cup (A \times C)$

Hence verified.

97. Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify that:

$A \times C \subset B \times D$

Ans. : We have,

A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}

$\therefore$ B × D = {1, 2, 3, 4} × {5, 6, 7, 8}

{(1, 5), (1, 6), (1, 7), (1, 8) , (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)} ....(i)

and, A × C = (1, 2) × (5, 6)

= {(1, 5), (1, 6), (2, 5), (2, 6)} ....(ii)

Clearly from equation (i) and equation (ii), we get

$\mathrm{A \times C \subset B \times D}$

Hence verified.

98. If A = {2, 3}, B = {4, 5}, C = {5, 6}, find $\mathrm{A \times (B \cap C)}$, $\mathrm{A \times (B \cap C), (A \times B) \cup (A \times C)}$.

Ans. : We have,

$\mathrm{A} = \{2,3\}$, $\mathrm{B} = \{4,5\}$ and $\mathrm{C} = \{5,6\}$

$\therefore\ \mathrm{B \cup C} = \{4,5\} \cup \{5,6\} = \{4,5,6\}$

$\therefore\ \mathrm{A} \times \{\mathrm{B \cup C}\} = \{2,3\} \times \{4,5,6\}$

$= \{(2,4),(2,5),(2,6),(3,4),(3,5),(3,6)\}$

Now,

$\mathrm{B \cap C} = \{4,5\} \cap \{5,6\} = \{5\}$

$\therefore\ \mathrm{A \times (B \cap C)} = \{2,3\} \times \{5\}$

$\mathrm{A \times (B \cap C)} = \{(2,5),(3,5)\}$

Now,

$\mathrm{A \times B} = \{2,3\} \times \{4,5\}$

$= \{(2,4),(2,5),(3,4),(3,5)\}$

and $\mathrm{A \times C} = \{2,3\} \times \{5,6\}$

$= \{(2,5),(2,6),(3,5),(3,6)\}$

$\therefore\ \mathrm{(A \times B) \cup (A \times C)} = \{(2,4),(2,5),(2,6),(3,4),(3,5),(3,6)\}$

99. If A = {-1, 1}, find A × A × A.

Ans. : We have,

A = {-1, 1}

∴ A × A = {-1, 1} × {-1, 1}

= {(-1, -1), (-1, 1), (1, -1), (1, 1)}

∴ A × A × A = {-1, 1} × {(-1, -1), (-1, 1), (1, -1), (1, 1)}

= {(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1), (1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)}

100. Determine the domain and range of the relation R defined by:

$\mathrm{R} = \{(\mathrm{x}, \mathrm{x}, +5) : \mathrm{x} \in \{0,1,2,3,4,5\}\}$

Ans. : We have,

$\mathrm{R} = \{(\mathrm{x}, \mathrm{x}, +5) : \mathrm{x} \in \{0,1,2,3,4,5\}\}$

For the elements of the given sets, we find that

R = {(0, 5), (1, 6), (2, 7), (3, 8),(4, 9), (5, 10)}

Clearly, Domain (R) = {0, 1, 2, 3, 4, 5} and Range (R) = {5, 6, 7, 8, 9, 10}

101. Let f and g be two real functions defined by $\mathrm{f(x)} = \sqrt{\mathrm{x}+1}$ and $\mathrm{g(x)} = \sqrt{9-\mathrm{x}^2}$ Then describe the following functions:

g - f

Ans. : We have,

$f(x)=\sqrt{x+1}$ and $g(x)=\sqrt{9-x^2}$

We observe that $f(x)=\sqrt{x+1}$ is defined for all $x\geq -1$

So, domain $f=[-1,\infty]$

Clearly, $g(x)=\sqrt{9-x^2}$ is defined for

$9-x^2\geq 0$

$\Rightarrow x^2-9\leq 0$

$\Rightarrow x^2-3^2\leq 0$

$\Rightarrow x\in[-3,3]$

$\therefore\ \text{domain}(g)=[-3,3]$

Now,

$\text{domain}(f)\cap\text{domain}(g)$

$=[-1,\infty]\cap[-3,3]$

$=[-1,3]$

f - g : [-3] → R is given by (g - f)(x) = g(x) - g(x)

$=\sqrt{9-x^2}-\sqrt{x+1}$

102. Let f and g be two real functions defined by $f(x)=\sqrt{x+1}$ and $g(x)=\sqrt{9-x^2}$ Then describe the following functions:

f^2+7f

Ans. : We have,

$f(x)=\sqrt{x+1}$ and $g(x)=\sqrt{9-x^2}$

We observe that $f(x)=\sqrt{x+1}$ is defined for all $x\geq -1$

So, domain $f=[-1,\infty]$

Clearly, $g(x)=\sqrt{9-x^2}$ is defined for

$9-x^2\geq 0$

$\Rightarrow x^2-9\leq 0$

$\Rightarrow x^2-3^2\leq 0$

$\Rightarrow x\in[-3,3]$

$\therefore\ \text{domain}(g)=[-3,3]$

Now,

$\text{domain}(f)\cap\text{domain}(g)$

$=[-1,\infty]\cap[-3,3]$

$=[-1,3]$

$f^2+7f:[-1,\infty]\to R$ defined by $(f^2+7f)(x)=f^2(x)+7f(x)\ \ [\because D(f)=[-1,\infty]]$

$=\left(\sqrt{x+1}\right)^2+7\sqrt{x+1}$

$=x+1+7\sqrt{x+1}$

103. Let f and g be two real functions defined by $f(x)=\sqrt{x+1}$ and $g(x)=\sqrt{9-x^2}$ Then describe the following functions:

$\frac{5}{g}$

Ans. : We have,

$f(x)=\sqrt{x+1}$ and $g(x)=\sqrt{9-x^2}$

We observe that $f(x)=\sqrt{x+1}$ is defined for all $x\geq -1$

So, domain $f=[-1,\infty]$

Clearly, $g(x)=\sqrt{9-x^2}$ is defined for

$9-x^2\geq 0$

$\Rightarrow x^2-9\leq 0$

$\Rightarrow x^2-3^2\leq 0$

$\Rightarrow x\in[-3,3]$

$\therefore\ \text{domain}(g)=[-3,3]$

Now,

$\text{domain}(f)\cap\text{domain}(g)$

$=[-1,\infty]\cap[-3,3]$

$=[-1,3]$

We have,

$g(x)=\sqrt{9-x^2}$

$\therefore\ 9-x^2=0$

$\Rightarrow x^2-9=0$

$\Rightarrow (x-3)(x+3)=0$

$\Rightarrow x=\pm 3$

So, domain $\left(\frac{1}{g}\right)=[-3,3]-\{-3,3\}=(-3,3)$

$\therefore\ \frac{5}{g}=(-3,3)\rightarrow R$ defined by $\left(\frac{5}{g}\right)(x)=\frac{5}{\sqrt{9-x^2}}$

104. Let f and g be two real functions defined by $f(x)=\sqrt{x+1}$ and $g(x)=\sqrt{9-x^2}$ Then describe the following functions:

fg

Ans. : We have,

$f(x)=\sqrt{x+1}$ and $g(x)=\sqrt{9-x^2}$

We observe that $f(x)=\sqrt{x+1}$ is defined for all $x\geq -1$

So, domain $f=[-1,\infty]$

Clearly, $g(x)=\sqrt{9-x^2}$ is defined for

$9-x^2\geq 0$

$\Rightarrow x^2-9\leq 0$

$\Rightarrow x^2-3^2\leq 0$

$\Rightarrow x\in[-3,3]$

$\therefore$ domain$(g) = [-3, 3]$

Now,

$\text{domain}(f) \cap \text{domain}(g)$

$= [-1, \infty] \cap [-3, 3]$

$= [-1, 3]$

fg : [-1, 3] → R is given by (fg)(x) = f(x) × g(x)

$= \sqrt{x+1} \times \sqrt{9-x^2}$

$= \sqrt{9+9x-x^2-x^3}$

105. Let f and g be two real functions defined by $f(x) = \sqrt{x+1}$ and $g(x) = \sqrt{9-x^2}$ Then describe the following functions:

$\frac{f}{g}$

Ans. : We have,

$f(x) = \sqrt{x+1}$ and $g(x) = \sqrt{9-x^2}$

We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \geq -1$

So, domain $f = [-1, \infty]$

Clearly, $g(x) = \sqrt{9-x^2}$ is defined for

$9 - x^2 \geq 0$

$\Rightarrow x^2 - 9 \leq 0$

$\Rightarrow x^2 - 3^2 \leq 0$

$\Rightarrow x \in [-3, 3]$

$\therefore$ domain$(g) = [-3, 3]$

Now,

$\text{domain}(f) \cap \text{domain}(g)$

$= [-1, \infty] \cap [-3, 3]$

$= [-1, 3]$

We have,

$g(x) = \sqrt{9-x^2}$

$\therefore\ 9 - x^2 = 0$

$\Rightarrow x^2 - 9 = 0$

$\Rightarrow (x-3)(x+3) = 0$

$\Rightarrow x = \pm 3$

So, domain $\left(\frac{f}{g}\right) = [-1, 3] - [-3, 3] = [-1, 3]$

$\therefore\ \frac{f}{g} : [-1, 3] \to R$ is given by $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x+1}}{\sqrt{9-x^2}}$

106. If $f(x) = \log_e(1-x)$ and $g(x) = [x]$, then determine the following functions:

$\left(\frac{f}{g}\right)\left(\frac{1}{2}\right)$

Ans. : We have,
$f(x) = \log_e(1 - x)$
and $g(x) = [x]$
$f(x) = \log_e(1 - x)$ is defined, if 1 - x > 0
$\Rightarrow 1 > x$
$\Rightarrow x < 1$
$\Rightarrow x \in (-\infty, 1)$
$\therefore$ Domain(f) $= (-\infty, 1)$
$g(x) = [x]$ is defined for all $x \in R$
$\therefore$ Domain(g) $= R$
$\therefore$ Domain(f) $\cap$ R Domain(g) $= (-\infty, 1) \cap R$
$= (-\infty, 1)$
$\left(\frac{f}{g}\right)\left(\frac{1}{2}\right)$= does not exist.

107. Is g = {(1, 1), (2, 3), (3, 5), (4, 7)} a function? Justify. If this is described by the relation, $g(x) = \alpha x + \beta$, then what values should be assigned to α and β?

Ans. : Given that: g = {(1, 1), (2, 3), (3, 5), (4, 7)}
Since every element of the domain in this relation has unique image, so g is a function.
Now $g(x) = \alpha x + \beta$,
For (1, 1) g(1) $= \alpha(1) + \beta = 1 \Rightarrow \alpha + \beta = 1 \ldots (1)$
For (2, 3) g(2) $= \alpha(2) + \beta = 3 \Rightarrow 2\alpha + \beta = 3 \ldots (2)$
Solving eqn. (i) and (ii) we have
$\alpha = 2$ and $\beta = -1$. [Note: We can take any other two ordered pairs]
Hence, the value of $\alpha = 2$ and $\beta = -1$.

* **Case study based questions** **[8]**

108. Ordered Pairs The ordered pair of two elements a and 3 is denoted by (a, b) : a is first element (or first component) and d is second element (or second component). Two ordered pairs are equal if their corresponding elements are equal. ie. (a, b) = (c, d)
⇒ a = c and b = d
Cartesian Product of Two Sets For two non-empty sets A and B, the cartesian product A . B is the set of all ordered pairs of elements from sets Aand B. In symbolic form, it can be written as
$A \cdot B = \{(a, b) : a \in A, b \in B\}$
Based on the above topics, answer the following questions.
If (a - 3, 6 + 7) = (3, 7), then the value of aand d are:
6, 0
3, 7
7, 0

3, -7
If (x + 6, y - 2) = (0, 6), then the value of x and y are:
6, 8
-6, -8
-6, 8
6, -8
If (x + 2, 4) = (5, 2x + y), then the value of x and y are:
-3, 2
3, 2
-3, -2
Let A and B be two sets such that A . B consists of 6 elements. If three elements of A . B are (1, 4), (2, 6) and (3, 6), then
(A . B) = (B . A)
$(\mathrm{A} \cdot \mathrm{B}) \neq (\mathrm{B} \cdot \mathrm{A})$
A . B = {(1, 4), (1, 6), (2, 4)}
None of the above
If m(A . B) = 45, then n(A) cannot be
15
17
5
9

Ans. : 1

109. Method to Find the Sets When Cartesian Product is Given For finding these two sets, we write first element of each ordered pair in first set say A and corresponding second element in second set B (say). Number of Elements in Cartesian Product of Two Sets If there are p elements in set A and g elements in set B, then there will be pq elements in A . B i.e. if n(A) = p and n(B) = q, then n(A . B) = pq.

Based on the above two topic, answer the following questions.

i. If A . B = {(a, 1), (b, 3), (a, 3), (b, 1), (a, 2), (b, 2)}. Then, A and B are:
   a. {1, 3, 2}, {a, b}
   b. {a, b}, {1, 3}
   c. {a, b}, {1, 3, 2}
   d. None of these
ii. If the set A has 3 elements and set B has 4 elements, then the number of elements in A . B is:
   a. 3
   b. 4
   c. 7
   d. 12

iii. A and B are two sets given in such a way that A . B contains 6 elements. If three elements of A . B are (1, 3), (2, 5) and (3, 3), then A, B are:
  a. {1, 2, 3}, {3, 5}
  b. {3, 5,}, {1, 2, 3}
  c. {1, 2}, {3, 5}
  d. {1, 2, 3}, {5}

iv. The remaining elements of A . B in (iii) is:
  a. (5, 1), (3, 2), (3, 5)
  b. (1, 5), (2, 3), (3, 5)
  c. (1, 5), (3, 2), (5, 3)
  d. None of the above

v. The cartesian product P . P has 16 elements among which are found (a, 1) and (b, 2). Then, the set P is:
  a. {a, b}
  b. {1, 2}
  c. {a, b,1, 2}
  d. {0, b, 1, 2, 4}

Ans. :

i. (c) {a, b}, {1, 3, 2}

Solution:

Here, first element of each ordered pair of A . B gives the elements of set A and corresponding second element gives the elements of set B.

∴ A = {a, b} and B = {1, 3, 2}

Note We write each element only one time in set, if it occurs more than one time.

ii. (d) 12

Solution:

Given, n (A) = 3 and n (B) = 4.

∴ The number of elements in A . B is:

n(A . B) = n(A) . n(B) = 3 . 4 = 12

iii. (a) {1, 2, 3}, {3, 5}

Solution:

It is given that (1, 3), (2, 5) and (3, 3) are in A . B. It follows that 1, 2, 3 are elements of A and 3, 5 are elements of B.

∴ A = {1, 2, 3} and B = {3, 5}

iv. (b) (1, 5), (2, 3), (3, 5)

Solution:

∵ A = {1, 2, 3} and B = {3, 5}

∴ A = {1, 2, 3} and B = {3, 5}

= {(1, 3), (1, 5), (2 3), (2, 5), (3, 3), (3, 5)}

Hence, the remaining elements of (A . B) are (1, 5), (2, 3), (3, 5).

v. {a, b,1, 2}

Solution:

Given, n(P . P) = 16

⇒ n(P) . n(P) = 16

⇒ n(P) = 4 ....(i)

Now, as $(a,1) \in P \cdot P$

$\therefore a \in P$ and $1 \in P$

Again, $(b,2) \in P \cdot P$

$\therefore b \in P$ and $2 \in P$

$\Rightarrow a, b, 1, 2 \in P$

From Eq. (i), it is clear that P has exactly four elements.

----- The only way to do great work is to love what you do. -----